$\boxed{V, Y, Z, R, X, G, B, \mathrm{S}, \mathrm{P}, \mathrm{Q}\ [=]\ \mathrm{pu}}$

1 Admittance Matrix

$Y = Z^{-1} = (R + \jmath X)^{-1} = G + \jmath B$

$y_{ii} = \sum_n Y_n$ (into node)

$y_{ij} = \sum_n -Y_n$ (directly between nodes)

1.1 Kron Reduction

$$2 \times 2 : Y_{\text{bus}} = \jmath \left[\begin{array}{c|c} B & C \\ \hline D & E \end{array}\right] \qquad Y'_{\text{bus}} = \jmath(B - CDE^{-1})$$

$$N \times N : \sum_{i=1}^{N} \sum_{j=1}^{N} Y'_{ij} = Y_{ij} - Y_{ik} Y_{jk} Y_{kk}^{-1} \qquad i \neq k, j \neq k$$

2 Power Flow Analysis

$\mathrm{Q}_i = \Im(\mathrm{S}_i) \qquad \mathrm{P}_i = \Re(\mathrm{S}_i) \qquad \delta_{ij} \equiv \delta_i - \delta_j$

$$\mathrm{S}_i = |V_i| \sum_{j=1, j\neq i}^{N} |V_j| (\cos\delta_{ij} + \jmath \sin\delta_{ij})(G_{ij} - \jmath B_{ij})$$

$$\mathrm{P}_i = |V_i| \sum_{j=1, j\neq i}^{N} |V_j| (G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}) = \mathrm{P}_{\mathrm{G},i} - \mathrm{P}_{\mathrm{D},i}$$

$$\mathrm{Q}_i = |V_i| \sum_{j=1, j\neq i}^{N} |V_j| (G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}) = \mathrm{Q}_{\mathrm{G},i} - \mathrm{Q}_{\mathrm{D},i}$$

2.1 Gauss-Seidel Method

$$\mathrm{S}_i = V_i \left(\sum_{j=1, j\neq i}^{N} Y_{ij} V_j \right)^* \qquad \left(\frac{\mathrm{S}_i}{V_i}\right)^* = Y_{ii} V_i + \sum_{j=1, j\neq i}^{N} Y_{i,j} V_j$$

$$V_i^{(k+1)} = \frac{1}{Y_{ii}} \left[\left(\frac{\mathrm{S}_i}{V_i^{(k)}} \right)^* - \sum_{j=1, j\neq i}^{N} Y_{ij} V_j^{(k)} \right]$$

2.2 Newton-Raphson Method

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad \boldsymbol{J} = \begin{bmatrix} \frac{\partial f_1(\boldsymbol{x})}{\partial x_1} & \cdots & \frac{\partial f_1(\boldsymbol{x})}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m(\boldsymbol{x})}{\partial x_1} & \cdots & \frac{\partial f_m(\boldsymbol{x})}{\partial x_n} \end{bmatrix} \quad \boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} f_1(x_1) \\ \vdots \\ f_n(x_n) \end{bmatrix}$$

$$\Delta\boldsymbol{x} = \boldsymbol{x}^{(k+1)} - \boldsymbol{x}^{(k)} = -[\boldsymbol{J}(\boldsymbol{x}^{(k)})]^{-1} \boldsymbol{f}(\boldsymbol{x}^{(k)})$$

$$\mathrm{P}_{i,\text{[P-V]}} = \sum_{j=1, j\neq i}^{N} |V_i||V_j| (G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}) + |V_i|^2 G_{ii}$$

$$\mathrm{Q}_{i,\text{[P-Q]}} = \sum_{j=1, j\neq i}^{N} |V_i||V_j| (G_{ij}\cos\delta_{ij} - B_{ij}\cos\delta_{ij}) - |V_i|^2 B_{ii}$$

2.2.1 Partial Derivatives

$\frac{\partial \mathrm{P}_i}{\partial \delta_j} = |V_i||V_j|(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij})$

$\frac{\partial \mathrm{P}_i}{\partial \delta_i} = -\mathrm{Q}_i - |V_i|^2 B_{ii}$

$\frac{\partial \mathrm{P}_i}{\partial |V_j|} = |V_i|(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij})$

$\frac{\partial \mathrm{P}_i}{\partial |V_i|} = \frac{\mathrm{P}_i}{|V_i|} + |V_i| G_{ii}$

$\frac{\partial \mathrm{Q}_i}{\partial \delta_j} = -|V_i||V_j|(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij})$

$\frac{\partial \mathrm{Q}_i}{\partial \delta_i} = \mathrm{P}_i - |V_i|^2 G_{ii}$

$\frac{\partial \mathrm{Q}_i}{\partial |V_j|} = |V_i|(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij})$

$\frac{\partial \mathrm{Q}_i}{\partial |V_i|} = \frac{\mathrm{Q}_i}{|V_i|} - |V_i| B_{ii}$

$\boldsymbol{J_1}$ $\quad$ $\boldsymbol{J_2}$

$$\boldsymbol{J} = \left[\begin{array}{ccc|ccc} \frac{\partial \mathrm{P}_2}{\partial \delta_2} & \cdots & \frac{\partial \mathrm{P}_2}{\partial \delta_N} & \frac{\partial \mathrm{P}_2}{\partial |V_2|} & \cdots & \frac{\partial \mathrm{P}_2}{\partial |V_N|} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \mathrm{P}_N}{\partial \delta_2} & \cdots & \frac{\partial \mathrm{P}_N}{\partial \delta_N} & \frac{\partial \mathrm{P}_N}{\partial |V_2|} & \cdots & \frac{\partial \mathrm{P}_N}{\partial |V_N|} \\ \hline \frac{\partial \mathrm{Q}_2}{\partial \delta_2} & \cdots & \frac{\partial \mathrm{Q}_2}{\partial \delta_N} & \frac{\partial \mathrm{Q}_2}{\partial |V_2|} & \cdots & \frac{\partial \mathrm{Q}_2}{\partial |V_N|} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \mathrm{Q}_N}{\partial \delta_2} & \cdots & \frac{\partial \mathrm{Q}_N}{\partial \delta_N} & \frac{\partial \mathrm{Q}_N}{\partial |V_2|} & \cdots & \frac{\partial \mathrm{Q}_N}{\partial |V_N|} \end{array}\right]$$

$\boldsymbol{J_3}$ $\quad$ $\boldsymbol{J_4}$

2.3 Fast Decouple Load Flow Method

$$\{\sin\delta_{ij} \approx 0, G \approx 0\} : \boldsymbol{J} = \left[\begin{array}{c|c} \boldsymbol{J_1}(\boldsymbol{x}^{(k)}) & 0 \\ \hline 0 & \boldsymbol{J_4}(\boldsymbol{x}^{(k)}) \end{array}\right]$$

$$\boldsymbol{\delta}^{(k+1)} = \boldsymbol{\delta}^{(k)} - \boldsymbol{J_1}^{-1} \cdot \left(\frac{\Delta\boldsymbol{P}}{|V|}\right)^{(k)}$$

$$|\boldsymbol{V}|^{(k+1)} = |\boldsymbol{V}|^{(k)} - \boldsymbol{J_4}^{-1} \cdot \left(\frac{\Delta\boldsymbol{Q}}{|V|}\right)^{(k)}$$

2.4 DC Power Flow Method

$\{\mathrm{Q} \approx 0,\ |V| = 1.0,\ G_{\text{Ln}} \approx 0\} :\ \boldsymbol{\delta} = -\boldsymbol{B}^{-1}\boldsymbol{P}$

3 Short Circuit Analysis by Symmetrical Components Analysis

$$\boldsymbol{A} \equiv \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \qquad \alpha \equiv 1\angle 120^\circ$$

$\bar{\boldsymbol{I}}_{abc} = \boldsymbol{A} \cdot \boldsymbol{I}^{0+-}$

$\bar{\boldsymbol{V}}_{abc} = \boldsymbol{A} \cdot \boldsymbol{V}^{0+-}$

$\bar{\boldsymbol{Z}}_{abc} = \boldsymbol{A} \cdot \boldsymbol{Z}^{0+-} \cdot \boldsymbol{A}^{-1} \qquad \bar{\boldsymbol{Z}}^{0+-} = \boldsymbol{A}^{-1} \cdot \boldsymbol{Z}_{abc} \cdot \boldsymbol{A}$

3.1 3 Phase Fault

$V^0 = V^- = 0 \qquad I^0 = I^- = 0$

$\bar{\boldsymbol{V}}_{abc} = \bar{\boldsymbol{Z}}_{\text{fault}} \bar{\boldsymbol{I}}_{abc} \qquad \bar{\boldsymbol{I}}_{abc} = \boldsymbol{A} \cdot \bar{\boldsymbol{I}}^{0+-}$

3.2 Single Line to Ground Fault

$I^0 = I^+ = I^- = \frac{V^+}{Z^0 + Z^+ + Z^-}$

$V^0 = I^0 Z^0 \qquad V^+ = V - I^+ Z^+ \qquad V^- = I^- Z^-$

3.3 Line to Line Fault

$I^0 = 0 \qquad I^+ = -I^- = \frac{V^+}{Z^+ + Z^-}$

$V^0 = 0 \qquad V^+ = V^- = V^+ \cdot \frac{Z^-}{Z^+ + Z^-}$

3.4 Double Line to Ground Fault

$I^+ = \frac{V^+}{Z^+ + (Z^0 \| Z^-)} \qquad I^- = -I^+ \cdot \frac{Z^0}{Z^0 + Z^-} \qquad I^0 = -(I^+ + I^-)$

$V^0 = V^+ = V^-$

4 Synchronous Machines

$I_{M,\text{rotor}} \frac{\mathrm{d}^2\theta_{\text{rotor}}}{\mathrm{d}t^2} = \vec{\tau}_{\text{net}} = \vec{\tau}_{\text{mech}} - \vec{\tau}_{\text{elec}}$

$\frac{2H}{\omega^{\ominus}} \left(\frac{\vec{\omega}_{\text{rotor}}(t)}{\vec{\omega}^{\ominus}_{\text{rotor}}} \right) \frac{\mathrm{d}^2\delta(t)}{\mathrm{d}t^2} = \mathrm{P}_{\text{net}} = \mathrm{P}_{\text{mech}} - \mathrm{P}_{\text{elec}} \qquad H \equiv \frac{\frac{1}{2} I_{M,\text{rotor}} \omega^2_{\text{rotor}}}{S_{\text{rated}}}$

$X(t) = \frac{N_{\text{poles}}}{2} X_{\text{rotor}}(t) \qquad X \in \{\vec{\alpha}, \omega, \delta\} \qquad \omega^{\ominus} = \frac{N_{\text{poles}}}{2} \vec{\omega}^{\ominus}_{\text{rotor}}$

4.1 Transient Stability Model

$\bar{I} = \frac{|\mathrm{P}|}{|V_{\text{bus}}| \cdot \mathrm{pf}} \angle \left(-\cos^{-1} \mathrm{pf} \right) \qquad \mathcal{E}' = V_{\text{bus}} + \jmath X_{\text{eq}} I \qquad \mathrm{P}_{\mathcal{E}} = \frac{|\mathcal{E}'||V_{\text{bus}}|}{X_{\text{eq}}} \sin\delta$

References

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